

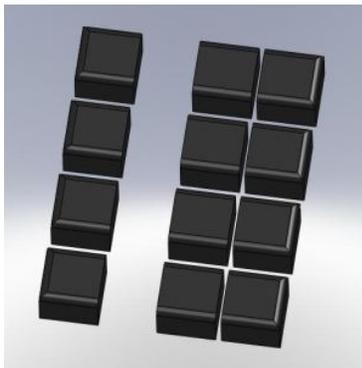
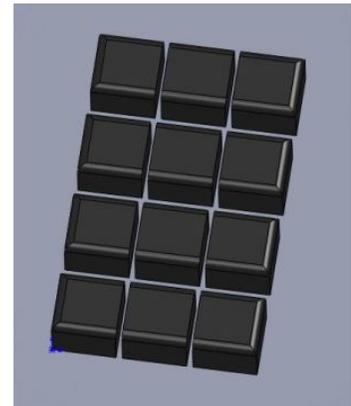
CCSSI # 5.NF.4a: Interpret the product  $(a/b) \times q$  as a parts of a partition of  $q$  into  $b$  equal parts; equivalently, as the result of a sequence of operations  $a \times q \div b$ . For example, use a visual fraction model to show  $(2/3) \times 4 = 8/3$ , and create a story context for this equation. Do the same with  $(2/3) \times (4/5) = 8/15$ . (In general,  $(a/b) \times (c/d) = ac/bd$ .)

The first request in this standard can be met with any problem that asks the question “what is two-thirds of four?” A story context might be something like this:



**Bruno went trick-or-treating, but his two brothers had the flu and had to stay home. When he got home found that he had four large Snickers bars in his bag. His mother told him that he had to share all of his candy equally with his ailing brothers. How should he divide the four Snickers bars and how many *bars* does he give to his brothers?**

Bruno could divide *each* of the four bars into *three equal pieces*. He then has twelve identical chunks of bar, each of which is one-third ( $1/3$ ) of a bar, or one-twelfth ( $1/12$ ) of the total.



Bruno chooses three so that the number of chunks he has now is a number that is divisible by three. Four of the twelve chunks represents one-third of the total that he keeps for himself. The other two-thirds that he gives to his brothers will then be eight chunks, or eight-thirds ( $8/3$ ) bars.

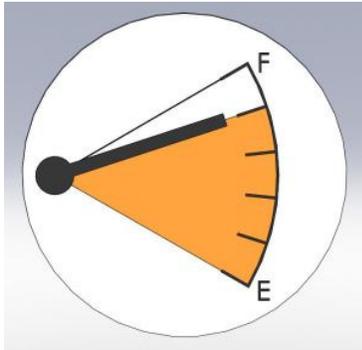
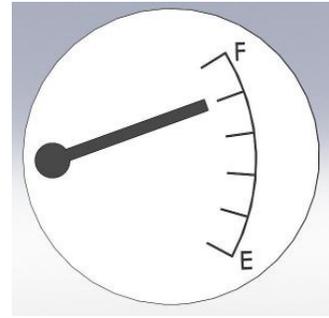
Word problems that involve multiplication of fractions generally use the word “of”, as in “one-half of such-and-such”. The word “of” can be replaced with the multiplication sign.

It is important to keep the context in mind throughout such problems, and represent it in the answer. In this case, we were asked “how many *bars*”, so a “whole” is one *bar*.

$$\frac{2}{3} \text{ ital of } 4 \text{ bars} = \frac{2}{3} \times 4 = \frac{2}{3} \times \frac{4}{1} = \frac{2 \times 4}{3 \times 1} = \frac{2 \times 4}{3} = \frac{8}{3} \text{ bars}$$

The second request in this standard asks for a context which requires the multiplication of two fractions. An example is as follows:

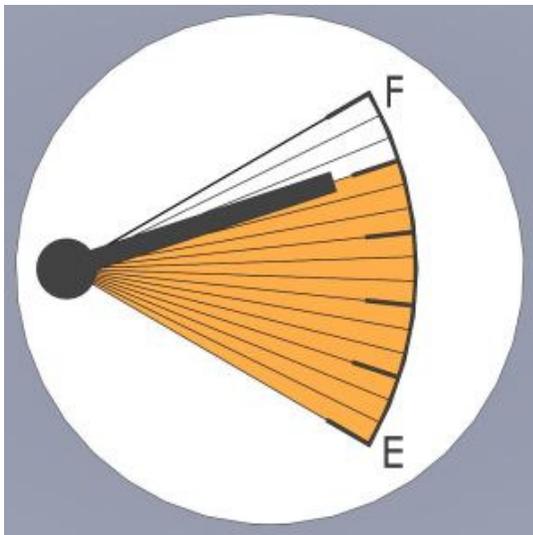
**The fuel gauge in Lilo's car told her that she had four fifths of a tank of gas. After a trip to the beach and back, she had used one third of the gas she started with. What fraction of a tank of gas did she have left?**



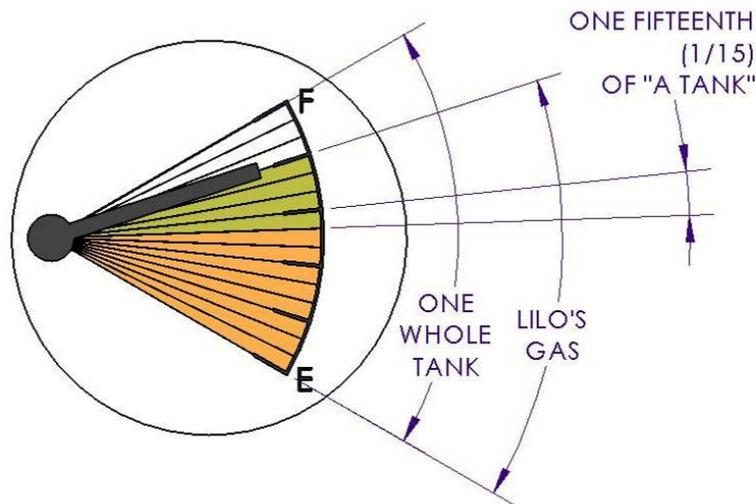
Her gas gauge is divided into five equal sections, or fifths, from empty (E) to full (F). The four sections below the needle represent four of the five fifths of the tank that contain gas before Lilo makes her trip to the beach. The upper fifth represents empty space in the tank.

If one third of the gas she started with was used, then she must have two thirds of what she started with left. So we need to find a way to represent the two-thirds of the gas she started with as a fraction of the capacity of the whole tank. Note that we have no information about how many gallons or liters the tank holds. The only unit we have to work with is tanks.

Since we want a visual representation of a fraction that is two-thirds ( $2/3$ ) of four-fifths ( $4/5$ ) of a tank, we will need to represent the gas that she has such that it can be divided into three equal portions.

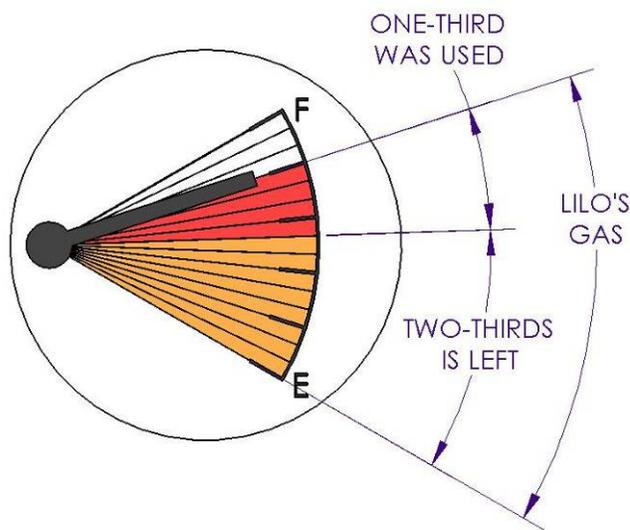


Imagine each *fifth* of the gas in her tank divided into *three* equal portions. There are now a total of fifteen equal portions, and each represents one-fifteenth of the whole tank. Three of those fifteenths represent empty space in the tank, but twelve of them represent the gas that Lilo starts out with.



$$\text{ONE FIFTEENTH (1/15) OF "A TANK"} = \text{ONE TWELFTH (1/12) OF "LILO'S GAS"}$$

Fractions are used to represent some portion of a "whole", but just what that "whole" represents is critical. Here we see how a given amount of gas can be represented as a different fraction, depending on whether it is used to represent a portion of a "tank", or of "Lilo's gas".



One-third of the twelve-fifteenths of the tank that are gas will be used making her round trip to the beach, and two-thirds of those twelve-fifteenths of the tank will remain in her tank. So here you can see how the remaining two-thirds (2/3) "of" Lilo's gas is the same as eight-fifteenths (8/15) "of" the tank.

Happily, it isn't necessary to draw detailed pictures to figure out fraction multiplication problems. As with the candy bar problem, we just replace the "of" with a multiplication operation.

$$\frac{2}{3} \text{ of } \frac{4}{5} \text{ tank} = \frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15} \text{ tank}$$